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Equating the values of c as given in (11) and (12),

$$\left(\frac{4x^2(3x-1)-2x(9x-1)(x+1)}{(9x+1)(x+1)}\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{y}{x+1}=0\ldots\ldots(13).$$

$$\therefore\ 2x(3x^2+10x-1)\frac{d^2y}{dx^2}-(9x+1)(x+1)\frac{dy}{dx}+(9x+1)y=0,$$

which is the Differential Equation given by the Praposer of the problem.

Scholium.—The proposed Differential Equation is satisfied by the equations, $y=1+x\ldots\ldots(\alpha)$ and $y=2x^{\frac{1}{2}}(3x-1\ldots\ldots(\beta)$; that is, these equations are particular solutions, or *particular integrals*, of the Differential Equation.

36. **Proposed by H. C. WHITAKER, B. Sc., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

A cube is revolved on its diagonal as an axis. Define the figure described and calculate its volume.

II. Solution by the PROPOSER.

The edges adjacent to the axis of revolution generate cones ; the two edges not adjacent to the axis generate a hyperboloid of one nappe. Take the axis of revolution as the axis of z. The equation of the cones is $x^2+y^2=2(z\pm\frac{1}{2}\sqrt{3}a)^2$, the altitude of each being $\frac{1}{3}\sqrt{3}a$, the radius of each being $\frac{1}{3}\sqrt{6}a$, the volume of each being $\frac{2\pi\sqrt{3}}{27}a^3$.

The equation of the hyperboloid is $2x^2+2y^2-4z^2=a^2$, the volume being the integral of $\pi(2z^2+\frac{a^2}{2})dz$ between the limits $\frac{1}{6}a\sqrt{3}$ and $-\frac{1}{6}a\sqrt{3}$, this volume being $\frac{5\pi\sqrt{3}}{27}a^3$. Adding the volume of the hyperboloid to the volume of the two cones, the total volume is found to be $\frac{\pi}{\sqrt{3}}a^3=1.8138a^3$.

[This solution is given for comparison with that of *DR. ZERR*, published in last issue. EDITOR.]

38. **Proposed by L. B. FILLMAN, St. Petersburg, Pennsylvania.**

The diameter of the circular base of a dome is $10=a$ feet, which is also the distance from any point on the circumference of the base to any point on the opposite side of the dome from base to apex. Find the volume of the dome.

I. Solution, by GEORGE B. McCLELLAN ZERR, M. A., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas; and the PROPOSER.

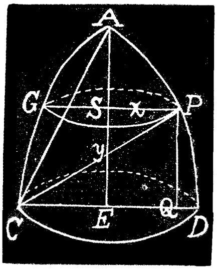
Let $x^2+y^2=a^2$ be the equation to the circle that forms a section of the dome perpendicular to the base. Then any section parallel to the base at distance y from the base has a radius$=(\sqrt{a^2-y^2}-\frac{1}{2}a)$.

$$\therefore\ v=\pi\int_0^{\frac{1}{2}a\sqrt{3}}\left(\sqrt{a^2-y^2}-\tfrac{1}{2}a\right)^2dy=\frac{\pi a^3}{24}(9\sqrt{3}-4\pi),$$

$$=\frac{125\pi}{3}(9\sqrt{3}-4\pi),\text{ when } a=10,$$

$$=395.59 \text{ cubic feet.}$$

II. Solution by Professor **J. SCHEFFER**, A. M., Hagerstown, Maryland; and **P. S. BERG**, Larimore, North Dakota.



$CD=CP=CA=2a$; $AE=\sqrt{4a^2-a^2}=a\sqrt{3}$. Let circular section PG be at distance $ES=y$, $PS=x$; then $(a+x)^2+y^2=4a^2$. $\therefore\ x=-a+\sqrt{4a^2-y^2}$.

$$\text{Capacity}=\pi\int_0^{a\sqrt{3}}x^2dy=\pi\int_0^{a\sqrt{3}}(5a^2-y^2-2a\sqrt{4a^2-y^2})dy$$

$$=\frac{\pi a^3}{3}(9\sqrt{3}-4\pi),\text{ and for } a=5, \frac{125\pi}{3}(9\sqrt{3}-4\pi).$$

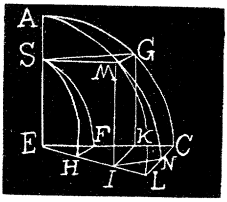
III. Solution by **FRANKLIN PIERCE MATZ**, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Let $AC=PC=DC=r,=10$ feet; $\angle ACD=\phi,\ =60^\circ=\frac{1}{3}\pi$; $\angle PCD=\theta$; $EQ=SP=x$; and $PQ=SE=y$. By well-known principles, then, we have $CE=r\cos\phi$; and, therefore, from the right-angled triangle CPQ, since $CQ=(r\cos\phi+x)$, we obtain the equation $(r\cos\phi+x)^2+y^2=r^2$ ..... (1). $\therefore\ x^2=r^2(\cos\theta-\cos\phi)^2$ ......(2). Now, $y=r\sin\theta$. $\therefore\ dy=r\cos\theta d\theta$......(3); also, from Calculus, $V=\pi\int x^2dy$......(4). Transforming (4) by means of (2) and (3), we obtain

$$V=\pi r^3\int_0^{\phi}(\cos\theta-\cos\phi)^2\cos\theta d\theta=\tfrac{1}{12}\pi r^3(\sin3\phi+9\sin\phi-12\phi\cos\phi)\ ......(5),$$

which is undoubtedly the *simplest* general result obtainable. Reducing (5) under the supposition that $\phi=60^\circ=\frac{1}{3}\pi$, we have the well-known result $V=\frac{1}{24}\pi(9\sqrt{3}-4\pi)r^3,=395.59+$ cubic feet.

IV. Solution by **W. L. HARVEY**, 75 Gray Street, Portland, Maine.



The arc of the dome$=60^\circ$ with radius$=10=a$; hence, the sine$=5\sqrt{3}=\frac{1}{2}a\sqrt{3}$, and this arc revolved about this sine generates the dome.

The quadrant CEA, revolving about EA, generates a hemi-sphere; ESF is a semi-segment of radius $=EA$. If the quadrant revolve a small distance, C

moving to L and G falling on M, and generating $CEALE$, and through MI a plane be passed parallel to CEA, the semi-segment $CGK=EFS$ generates $CKILMG$, of which the part $INLM=EFHS$.

The volume generated by the semi-segment EFS in an entire revolution will equal that generated by CGK *minus* the sum of the solids $CKNIGM$ lying about the circumference of the base of the hemi-sphere. But $GMCKIN=KI\times$ area of the semi-segment CGK; and the sum of all these parts is equal to the circumference described by EK as radius into the same area. If $EA=a$, $EI=c$, $ES=s$, and the arc $SF=p$, we obtain for the solid generated by $EAGK$, $\frac{2\pi}{3}(sc^2+a^3-a^2s)$. Consequently the solid generated by $CGK=\frac{2\pi}{3}(a^2s-sc^2)$. Then the sum of all the solids $CNIKMG=$semi-segment $GCK\times 2\pi c=\pi(cap-sc^2)$, and the volume sought is $\frac{2\pi}{3}(a^2s-sc^2)-\pi(cap-sc^2)=\pi(\frac{sc^2}{3}+\frac{2a^2s}{3}-cap)$. Putting $c^2=a^2-s^2$ this becomes, $\pi(sa^2-\frac{s^3}{3}-cap)$. In the problem $s=\frac{a}{2}\sqrt{3}$, $c=\frac{a}{2}$, $p=a(60^\circ)=\frac{a\pi}{3}$. Then $cap=\frac{a^3\pi}{2.3}$, and the contents are $\frac{a^3\pi}{2}\left(\frac{3\sqrt{3}}{4}-\frac{\pi}{3}\right)=395.59$ cubic feet. This method of solution was suggested by a solution of a similar problem by Professor Seyford, of Colby University.

[From the MONTHLY of October, 1894, pp. 257-8, we have *four* other different solutions of a similar problem; and *each* solution gives the result, $V=\frac{1}{3}\pi(9\sqrt{3}-4\pi)a^3$, $\frac{125}{3}\pi(9\sqrt{3}-4\pi)=395.59027+$ cubic feet. EDITOR.]

39. Proposed by J. C. GREGG, Brazil, Indiana.

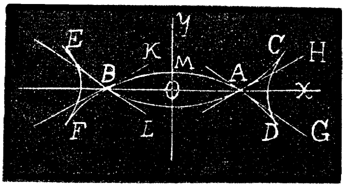
Show that the curve

$$x=9a\sin\theta-4a\sin^3\theta$$
$$y=-3a\cos\theta+4a\cos^3\theta$$

is symmetrical to the axes, and has double points and cusps: find the lengths of the arcs, (a) between the double points, (b) between a double point and a cusp, (c) and the arc connecting two cusps and not passing through a double point. [*Johnson's Calculus.*]

Solution by GEORGE B. McCLELLAN ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

The equations as given in *Johnson's Calculus* are



$x=9a\sin\theta-4a\sin^3\theta=6a\sin\theta+a\sin3\theta$,
$y=-3a\cos\theta+4a\cos^3\theta=a\cos3\theta$.
$\therefore r^2=x^2+y^2=a^2+72a^2\sin^2\theta-48a^2\sin4\theta$;
$dx=6a\cos\theta+3a\cos3\theta$, $dy=-3a\sin3\theta$.
$ds=\sqrt{dx^2+dy^2}=3a(4\cos^2\theta-1)$.
$\therefore s=3a(\theta+\sin2\theta)$.